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Ancon

Shearfix Punching Shear Reinforcement

Design Manual to ACI 318-19

Ancon Shearfix Design Manual: ACI 318-19

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Introduction

The Ancon Shearfix software from Leviat supports engineers in the design of punching shear situations. It helps to calculate the punching strength without shear reinforcement and provides a solution with Shearfix studs if shear reinforcement is necessary. After the design, one can directly create a parts list for the specification or a dxf file to import the solution into a CAD drawing. Additionally, the output report provides a detailed verification of the proposed solution so that the engineer has the opportunity to check and verify the calculation performed by the software.

Since punching can be a rather brittle failure and thus can lead to severe damage, engineers should take sufficient care in the design of such situations. Thus, users should not only operate the design software but also understand the calculation behind it. This document provides the software users with the necessary background information to follow the design. Additionally, it highlights certain aspects of the general punching shear design and provides additional information.

This document is based on Leviat's longstanding knowledge of punching shear design and experience of manufacturing a wide range of punching shear reinforcement systems in numerous countries. Leviat has performed dozens of punching shear tests over many years meaning engineers all over the world can use this guide, and specify punching shear reinforcement, with confidence.

Calculation of factored shear stress

Lateral loading and unbalanced gravity load can lead to a transfer of moment between the slab and the column. This moment transfer is linked to a non-uniform distribution of -factored- shear stress along the critical section. Thus, for the verification, one requires the maximum shear stress at the critical section. The maximum shear stress v_u is obtained by :

$$v_u = \frac{V_u}{b_0 d} - \frac{\gamma_{vx} \cdot M_{ux} \cdot y_{vu,max}}{J_x} + \frac{\gamma_{vy} \cdot M_{uy} \cdot x_{vu,max}}{J_y} \qquad \text{Eq. 1}$$

where V_u is the punching shear load, b_0 is the length of the critical section, d is the effective depth, γ_{vx} and γ_{vy} are factors to determine the proportion of the slab moment transferred by eccentricity of shear at slab-column connections, $x_{vu,max}$ and $y_{vu,max}$ are the coordinates of the location of the maximum shear stress, and J_x and J_y are properties of the assumed critical section analogous to polar moments of inertia (see chapter polar moments / moments of inertia).

Since $x_{vu,max}$ and $y_{vu,max}$ are unknown initially, the shear stress needs to be calculated at various locations along the critical section to find the location, and thus the value, of the maximum shear stress v_u. It has to be noted that the coordinates x and y refer to the coordinate system aligned with the principal axes.

The factors γ_{vx} and γ_{vy} used in the Shearfix Software are determined according to ACI 421.1R-20 Appendix B.

Interior slab-column connections:

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_y/l_x}} \quad \text{Eq. 2a}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_x/l_y}} \quad \text{Eq. 2b}$$

Edge slab-column connections:

$$\gamma_{vx} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{l_y/l_x}} \quad \text{Eq. 3a}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{l_x}{l_y} - 0.2}} \text{ but } \gamma_{vy} = 0 \text{ if } \frac{l_x}{l_y} < 0.2 \quad \text{Eq. 3b}$$

Corner slab-column connections:

$$\gamma_{vx} = 0.4 \quad \text{Eq. 4a}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{\frac{l_x}{l_y} - 0.2}} \text{ but } \gamma_{vy} = 0 \text{ if } \frac{l_x}{l_y} < 0.2 \quad \text{Eq. 4b}$$

l_x and l_y are projections of the shear critical section in the direction of the principal x- and y-axes.

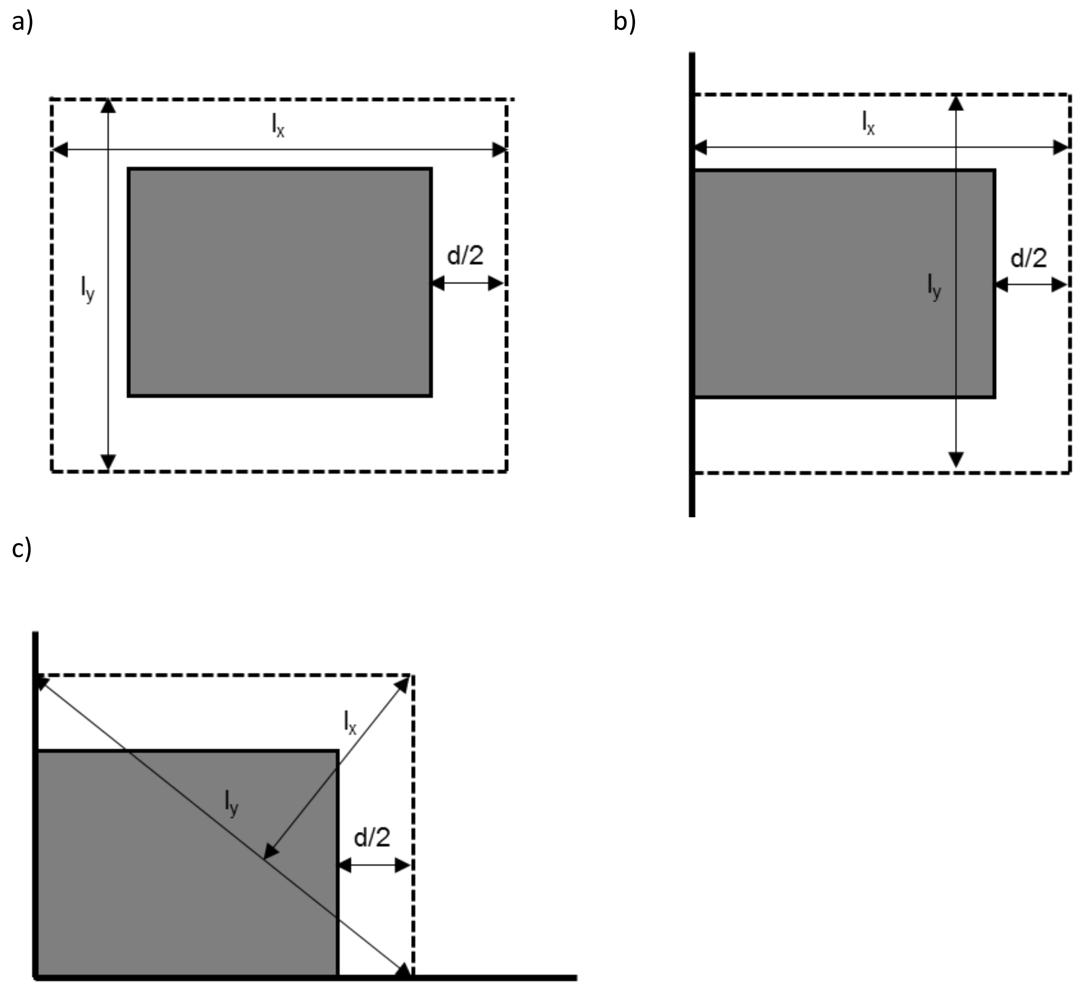

Figure 1: Definition of lx and ly for (a) interior slab-column connections, (b) edge slab-column connections, and (c) corner slab-column connections

If punching shear reinforcement is required, the factored shear stress needs to be calculated at an outer critical section also. For these cases equation 1 becomes

$$v_u = \frac{V_u}{b_{out}d} - \frac{\gamma_{vx} \cdot M_{ux} \cdot y_{vu,max}}{J_x} + \frac{\gamma_{vy} \cdot M_{uy} \cdot x_{vu,max}}{J_y} \qquad \text{Eq. 5}$$

where V_u is the punching shear load, b_{out} is the length of the critical section outside the shear-reinforced area, d is the effective depth, γ_{vx} and γ_{vy} are factors to determine the proportion of the slab moment transferred by eccentricity of shear at slab-column connections, $x_{vu,max}$ and $y_{vu,max}$ are the coordinates of the location of the maximum shear stress, and J_x and J_y are properties of the assumed critical section analogous to polar moments of inertia (see chapter polar moments / moments of inertia)

The factors γ_{vx} and γ_{vy} are taken as shown previously (Eq. 2 to Eq. 4). However, the distances l_x and l_y refer to the critical section outside the shear-reinforced area.

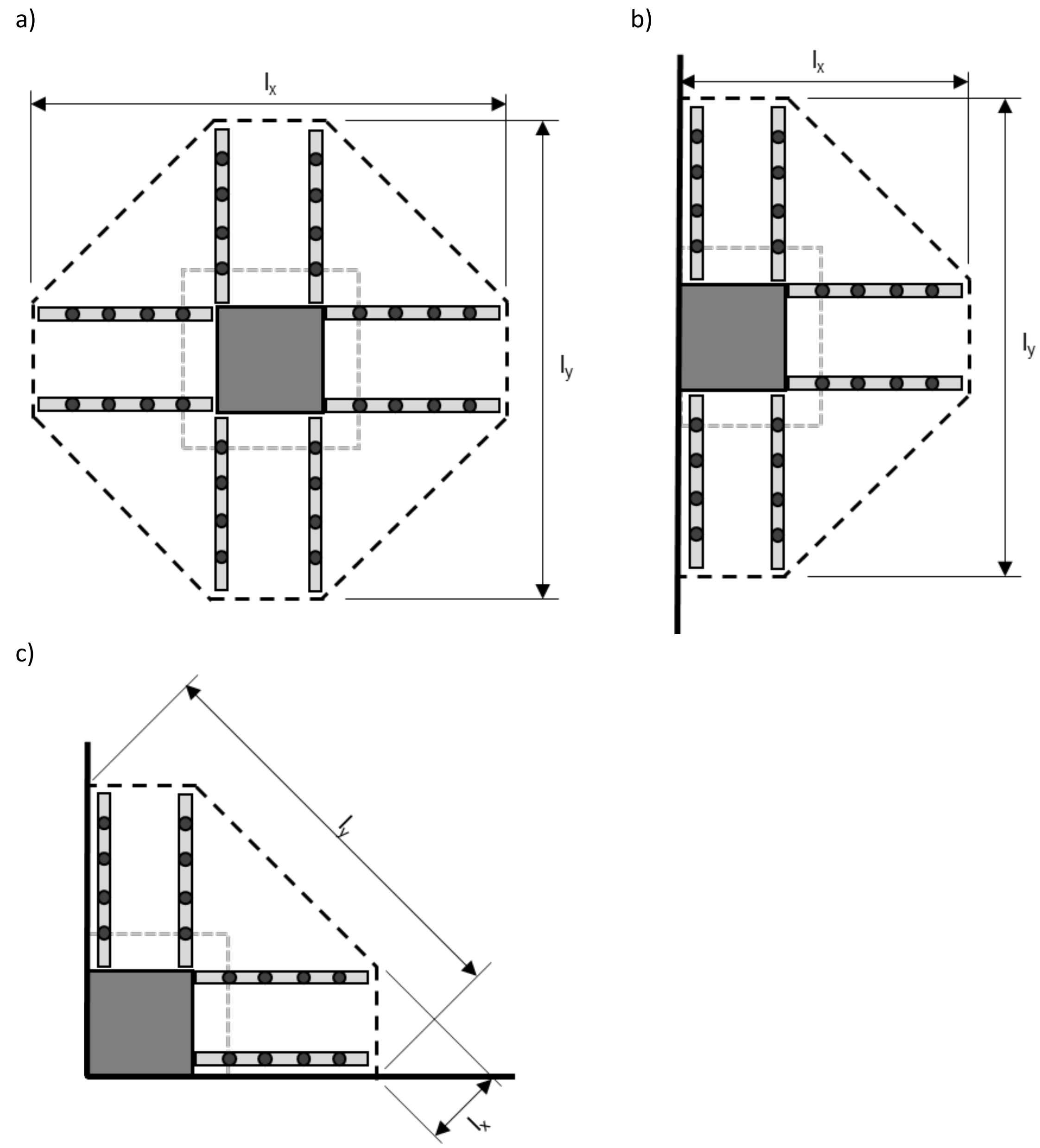

Figure 2: Definition of lx and ly at the critical section outside the shear-reinforced area for (a) interior slab-column connections, (b) edge slab-column connections, and (c) corner slab-column connections

Punching strength without shear reinforcement

Generally, design codes define the punching strength without shear reinforcement by assuming a concrete shear strength along a critical section defined by the effective depth multiplied by the length of the critical section. If the punching strength is larger than the design forces, the design is verified. However, only a few design codes include the deformation capacity related to the punching of flat slabs. Punching shear failure without shear reinforcement can be rather brittle, thus failure occurs at rather small deformations without any warning signs and without any possibility of load distribution to other supports. Consequently, the robustness of the overall structure can be limited by these column-slab connections. An increase in the robustness can be achieved by using shear reinforcement since it does not only increase the punching strength but also the deformation capacity. In fact, by adding shear reinforcement, the increase in the deformation capacity is greater than the increase in the punching strength. Therefore, in certain situations or special buildings, it may be recommended to use shear reinforcement even when the punching strength without shear reinforcement is larger than the design load in order to increase the robustness of the overall structure.

Regarding the punching strength design according to ACI 318, the concrete shear strength for non-prestressed slabs and footings is defined by the minimum of the following values:

$$v_c = 0.33\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 6a}$$

$$v_c = 0.17\left(1+\frac{2}{\beta}\right)\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 6b}$$

$$v_c = 0.083\left(2+\frac{\alpha_s d}{b_0}\right)\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 6c}$$

where λ_s is a size effect modification factor, λ is a reduction factor for light-weight concretes, β is the aspect ratio of the column in the case of rectangular columns, f'_c is the specified compressive strength of the concrete, b_0 is the length of the critical section, d is the effective depth, and α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns.

The size effect factor considers the less-than-proportional increase in the shear strength with increasing slab thickness. It is calculated as follows:

$$\lambda_s = \sqrt{\frac{2}{1+\frac{d}{254}}} \leq 1.0 \qquad \text{Eq.7}$$

Design safety is fulfilled if the punching shear strength multiplied by the strength reduction factor ϕ ($\phi = 0.7$) is larger than the design shear stress v_u.

$$v_u \leq \phi v_c \qquad \text{Eq. 8}$$

Punching strength with shear reinforcement

If shear reinforcement is present, one can generally distinguish between three punching failure modes for the design of flat slabs:

- Failure of the compression strut next to the column
- Failure within the shear-reinforced area
- Failure outside the shear-reinforced area

The latter two failures can be avoided by installing more shear reinforcement – by increasing the cross-sectional area of shear reinforcement inside the critical section or by increasing the size of the shear-reinforced area, respectively. Failure of the compression strut however is not influenced by the amount of shear reinforcement but rather by its location - accounted for by the detailing rules in design codes - and the anchorage performance of the shear reinforcement as well as the geometrical boundary conditions such as the column size and the slab thickness. Therefore, the compression failure is often called the maximum punching strength since it is an upper limit for a certain design situation.

The punching strength against failure of the compression strut for shear stud reinforcement is given by

$$v_{c,max} = 0.66\sqrt{f_c'} \qquad \text{Eq. 9}$$

It can be noted that ACI 318 permits a larger upper limit for shear studs than for stirrups. However this larger limit can only be considered if the spacing between the studs is $s \leq 0.5d$ since the experimental data used as a basis for the punching shear provisions in ACI 318 did not provide sufficient data for larger spacing. Thus, if the spacing between the studs is $s > 0.5d$, the maximum punching strength is limited to the same value as for stirrups:

$$v_{c,max} = 0.5\sqrt{f_c'} \qquad \text{Eq. 10}$$

Design safety is fulfilled if the punching shear strength multiplied by the strength reduction factor ϕ ($\phi = 0.75$) is larger than the design shear stress v_u

$$v_u \leq \phi v_{c,max} \qquad \text{Eq. 11}$$

Regarding failure within the shear-reinforced area, ACI 318 basically considers a summation of the punching shear strength of the concrete and the shear reinforcement. However, the concrete strength is reduced compared to the verification of slabs without shear reinforcement. The reason for this is that if shear reinforcement is used, higher load levels are achieved leading to larger slab deformations and thus to larger openings of shear cracks within the slab, which results in a decline of the concrete shear strength (refer to commentary in ACI 318). Thus, ACI 318 uses only three quarters of the concrete shear strength without shear reinforcement if double headed studs are used. The concrete shear strength within the shear-reinforced area v_{cs} is defined by the minimum of the following values:

$$v_{cs} = 0.25\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 12a}$$

$$v_{cs} = 0.17\left(1 + \frac{2}{\beta}\right)\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 12b}$$

$$v_{cs} = 0.083\left(2 + \frac{\alpha_s d}{b_0}\right)\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 12c}$$

where λ_s is a size effect modification factor, λ is a reduction factor for light-weight concretes, β is the aspect ratio of the column in the case of rectangular columns, f'_c is the specified compressive strength of the concrete, b_0 is the length of the critical section, d is the effective depth, and α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns.

The contribution of the shear reinforcement is given by:

$$v_s = \frac{A_v \cdot f_{yt}}{b_0 \cdot s} \qquad \text{Eq. 13}$$

where A_v is the sum of the cross sectional area of all shear stud reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section, f_{yt} is the yielding strength of the shear stud reinforcement, b_0 is the length of the critical section, and s is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

It has to be noted that ACI 318 requires a minimum amount of shear reinforcement if shear studs are used:

$$\frac{A_v}{s} \geq 0.17\sqrt{f_c'}\frac{b_0}{f_{yt}} \qquad \text{Eq. 14}$$

Design safety is fulfilled if the punching shear strength multiplied by the strength reduction factor ϕ ($\phi = 0.75$) is larger than the design shear stress v_u

$$v_u \leq \phi(v_{cs} + v_s) \qquad \text{Eq. 15}$$

Regarding the punching strength outside the shear-reinforced area, one has to verify that the shear strength of the slab is sufficient at an outer critical section. However, compared to the verification of the punching shear strength without shear reinforcement at the column face, a reduced concrete shear strength is taken for the verification of the punching shear strength outside the shear-reinforced area leading to the following expression:

$$v_{c,out} = 0.17\lambda_s\lambda\sqrt{f_c'} \qquad \text{Eq. 16}$$

Design safety is fulfilled if the punching shear strength multiplied by the strength reduction factor ϕ (ϕ = 0.75) is larger than the design shear stress v_u

$$v_{u,out} \leq \phi v_{c,out} \qquad \text{Eq. 17}$$

Critical section at the column

The critical section is set at a distance of d/2 from the column face. Figure 3 shows this critical section for a rectangular and a circular column. The critical sections shown are valid for interior columns without nearby openings. If openings or slab edges are present, the critical section must be adjusted accordingly.

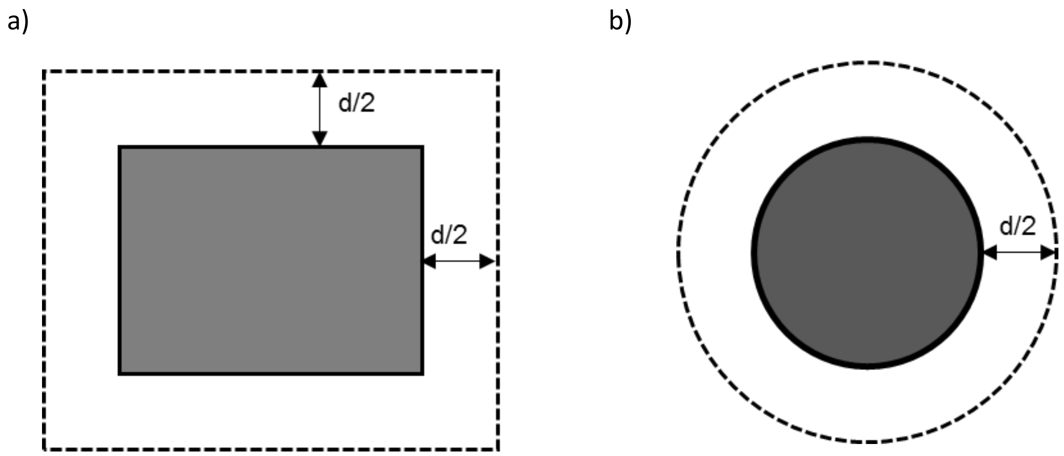

Figure 3: Critical section at the column for interior columns without openings

Openings must be considered in the design if they are closer than a distance of 4h from the column face, where h is the slab thickness. In such cases, the critical section is reduced by an ineffective section defined by two tangential lines starting at the center of the column (refer to figure 4).

a) b)

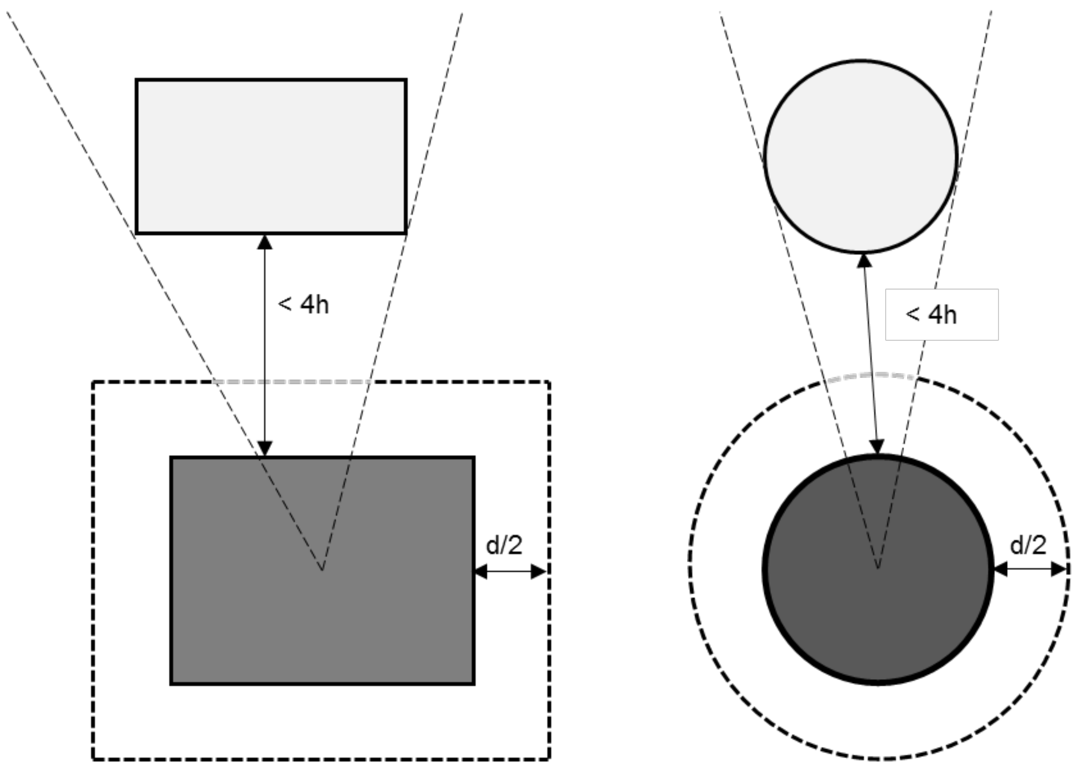

Figure 4: Critical section at the column for interior columns with openings

It has to be noted that the Shearfix software considers all the openings if they are defined by the user even if they are further away than 4h from the column face. The reason for this is that although they may not have to be considered for the verification at the column face, they may affect the verification of the shear reinforcement or the outer critical section. However, the software will display an information message if an opening is further than 4h from the column face so that users can decide for themselves if the opening should be kept or deleted.

In the case of a column close to a slab edge, i.e. edge column, corner column, or re-entrant corner column, the critical section is taken as shown in Figure 3, 5 or 6 – whichever gives the smallest perimeter.

a) b)

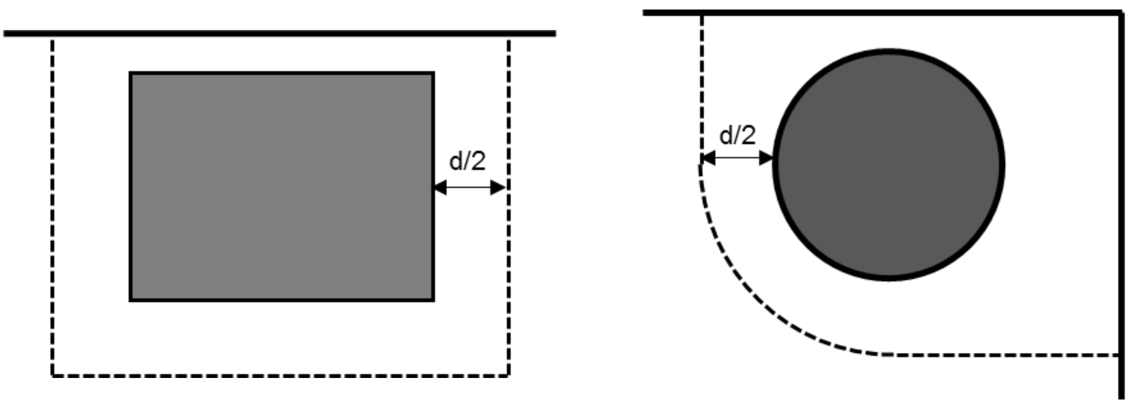

Figure 5: Critical section at the column for (a) an edge or (b) a corner column

In the case of a re-entrant corner, several possible critical sections can be assumed. The Shearfix software distinguishes three possibilities: either the critical section runs perpendicular to the slab edge (figure 6a), it connects to the corner of the two slab edges (figure 6b), or it uses the continuous critical section (figure 3). Thus, depending on the location, shape, and orientation of the column, multiple different critical sections can be defined. The Shearfix software always uses the smallest one that can be found.

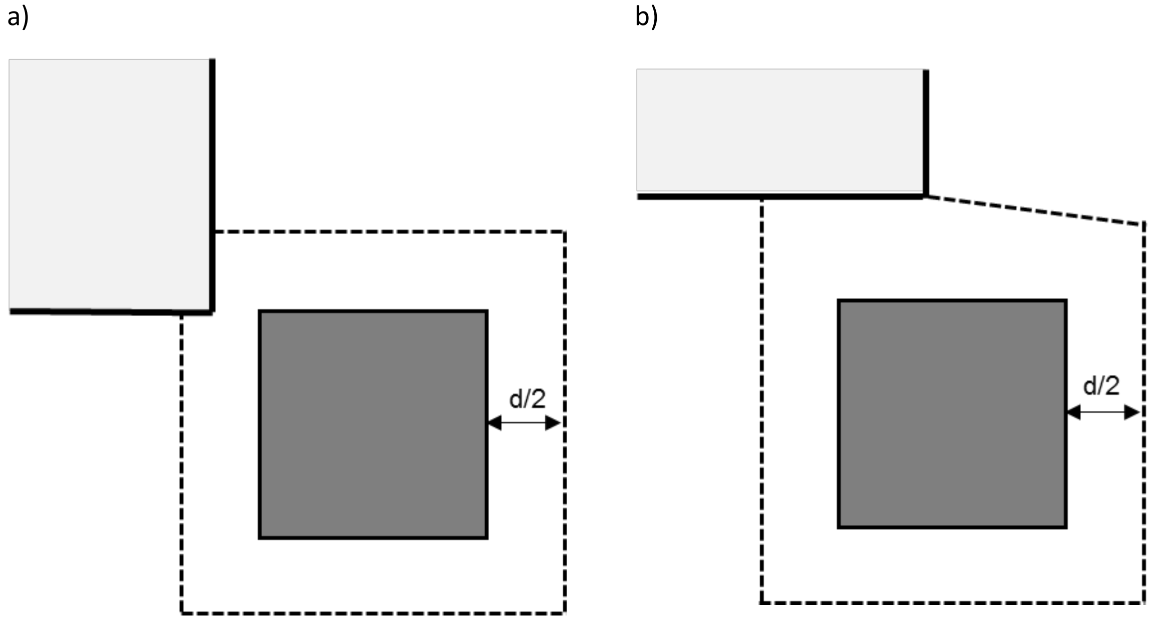

Figure 6: Critical section in the case of a re-entrant corner

Regarding the critical section in the case of openings and columns close to slab edges, one has to note that an inconsistency exists in the definition of the critical section. For example, if a large opening is defined, the critical section might be smaller than if the situation is designed as an edge column. For these cases, ACI 318 recommends the use of the edge column critical section. However, even this recommendation does not rule out all the inconsistencies. Thus, it is up to the user, to carefully choose the correct and safe design situation.

Critical section outside the shear-reinforced area

The critical section for the verification outside the shear-reinforced area is located at a distance d/2 beyond the outermost shear stud of each rail (refer to figure 7). If openings and slab edges are present, the critical section outside the shear-reinforced area is adjusted in a similar manner as the critical section at the column (refer to figure 4, 5, and 6).

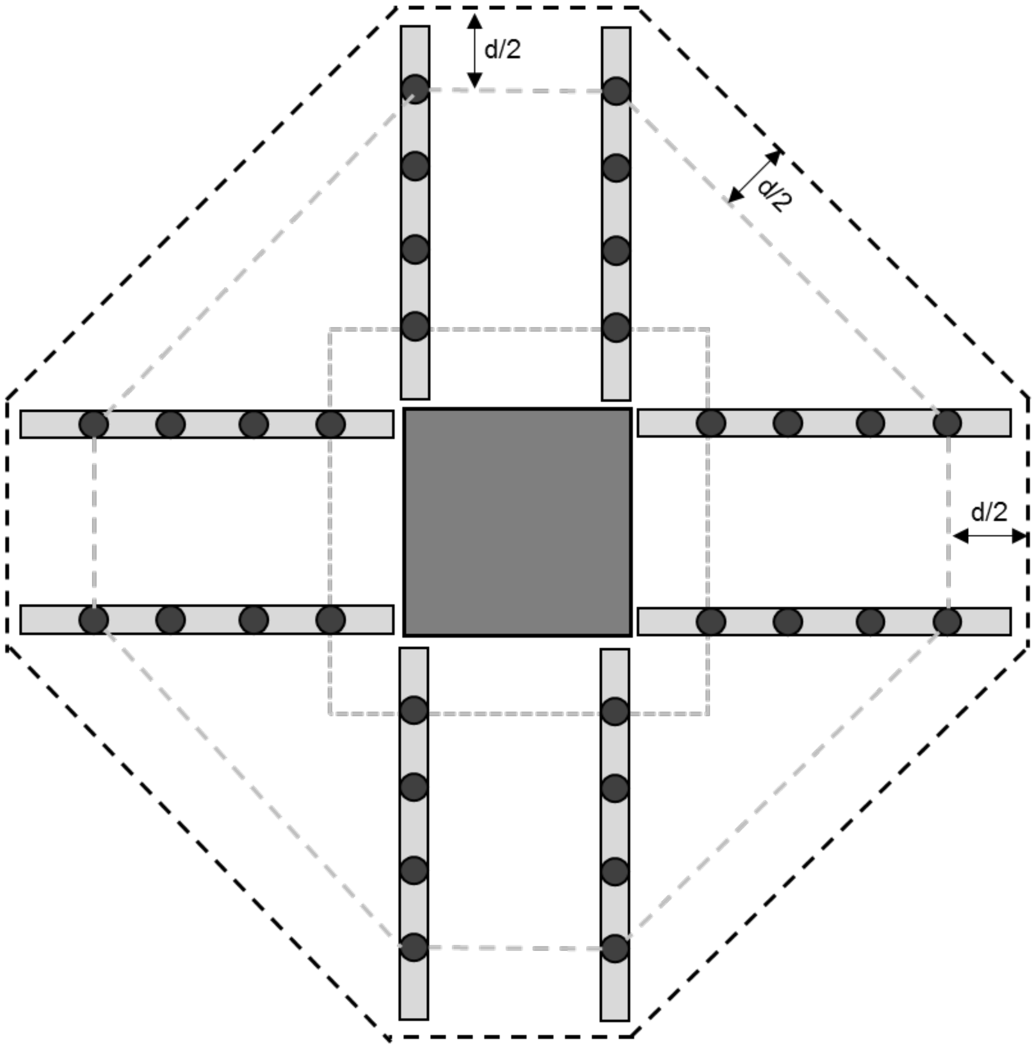

Figure 7: Critical section outside the shear reinforcement area

Polar moment / Moment of inertia

For the calculation of the factored shear stresses, ACI 318 uses the value J_c which is defined as a property of the assumed critical section analogous to a polar moment of inertia. ACI 318 provides the formulation for J_c for an interior slab column connection:

$$J_c = \frac{d(c_1 + d)^3}{6} + \frac{d^3(c_1 + d)}{6} + \frac{d(c_2 + d)(c_1 + d)^2}{2} \qquad \text{Eq. 18}$$

where d is the effective depth, c_1 is the longer dimension of a rectangular column, and c_2 is the shorter length of a rectangular column.

For edge or corner slab column connections, one can find similar equations in textbooks. However, while for these three standard cases solutions for J_c exist, J_c is ill-defined for other cases - such as for circular columns and for situations with large openings next to the column. Additionally, since the critical section outside the shear-reinforced area contains diagonal sides, the calculation of the polar moment leads to difficulties. Because of these issues, the polar moment is often replaced by the moment of inertia of the critical section. Especially when programming universally valid cases, the use of the moment of inertia has advantages since it allows the use of a more general approach.

The calculation of the moment of inertia leaves out the second term in equation 18. Thus, it leads to slightly lower values which generally leads to conservative designs.

The moment of inertia for a straight segment is defined as

$$J_{x,i} = d \cdot \frac{l}{3} \cdot \left(y_i^2 + y_i y_{i+1} + y_{i+1}^2\right) \qquad \text{Eq. 19a}$$

$$J_{y,i} = d \cdot \frac{l}{3} \cdot \left(x_i^2 + x_i x_{i+1} + x_{i+1}^2\right) \qquad \text{Eq. 19b}$$

where d is the effective depth, l is the length of the segment, y_i and x_i are the coordinates of the start point, and x_{i+1} and y_{i+1} are the end point of the segment.

By summation of the individual segments, one can obtain the moment of inertia for the entire critical section.

$$J_x = d \sum \left[\frac{l}{3} \cdot \left(y_i^2 + y_i y_{i+1} + y_{i+1}^2\right)\right] \qquad \text{Eq. 20a}$$

$$J_y = d \sum \left[\frac{l}{3} \cdot \left(x_i^2 + x_i x_{i+1} + x_{i+1}^2\right)\right] \qquad \text{Eq. 20b}$$

J_x and J_y used in the calculation of the factored shear stresses are determined relative to the principal axes. If the critical section does not have an axis of symmetry, a rotation of the coordinate system is required (refer to figure 8).

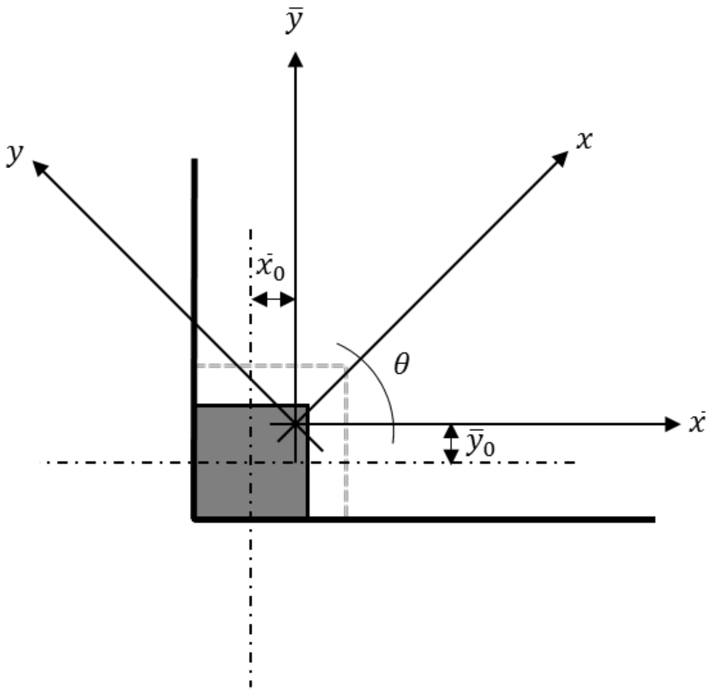

Figure 8: Definition of the coordinate systems in case of a corner slab-column connection

The direction of the principal axes is defined by the angle θ which can be determined by

$$J_{\bar{x}} > J_{\bar{y}} \qquad \tan 2\theta = \frac{-2J_{\overline{xy}}}{J_{\bar{x}} - J_{\bar{y}}} \qquad \text{Eq. 21a}$$

$$J_{\bar{x}} < J_{\bar{y}} \qquad \tan(2\theta - \pi) = \frac{-2J_{\overline{xy}}}{J_{\bar{x}} - J_{\bar{y}}} \text{ i} \qquad \text{Eq. 21b}$$

$$J_{\bar{x}} = J_{\bar{y}};\ -2J_{\overline{xy}} = 0 \qquad \theta = 0 \qquad \text{Eq. 21c}$$

$$J_{\bar{x}} = J_{\bar{y}};\ -2J_{\overline{xy}} < 0 \qquad \theta = -\frac{\pi}{4} \qquad \text{Eq. 21d}$$

$$J_{\bar{x}} = J_{\bar{y}};\ -2J_{\overline{xy}} > 0 \qquad \theta = \frac{\pi}{4} \qquad \text{Eq. 21e}$$

with

$$J_{\bar{x}} = d \sum \left[\frac{l}{3} \cdot \left(\bar{y}_i^2 + \bar{y}_i \bar{y}_{i+1} + \bar{y}_{i+1}^2\right)\right] \qquad \text{Eq. 22a}$$

$$J_{\bar{y}} = d \sum \left[\frac{l}{3} \cdot \left(\bar{x}_i^2 + \bar{x}_i \bar{x}_{i+1} + \bar{x}_{i+1}^2\right)\right] \qquad \text{Eq. 22b}$$

$$J_{\overline{xy}} = d \sum \left[\frac{l}{6} \cdot \left(2\bar{x}_i \bar{y}_i + \bar{x}_i \bar{y}_{i+1} + \bar{x}_{i+1} \bar{y}_i + 2\bar{x}_{i+1} \bar{y}_{i+1}\right)\right] \qquad \text{Eq. 22c}$$

where $\bar{x}$ and $\bar{y}$ are the coordinates in non-principal directions.

It has to be noted that Equations 19 to 22 are valid for a coordinate system with its origin at the center of the critical section. Thus, moments acting at the center of the column may have to be adjusted to consider an eventual offset between the center of the column and the center of the critical section (distances $\bar{x}_0$ and $\bar{y}_0$ in figure 8). The Shearfix software does this automatically, so that the user can simply enter the moments acting at the center of the columns.

Prestressing

Prestressing has a beneficial effect on the punching strength. The compressive stresses due to prestressing reduce the cracking of the concrete and thus increase the concrete shear strength. Additionally, any inclination of the prestressing tendons at the critical section help resist the shear load. ACI 318 allows consideration of both of these effects with the parameters f_{pc} and V_p in Equations 23a and 23b, which define the concrete shear strength of prestressed slabs.

$$v_c = 0.29\lambda\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b_0 d} \qquad \text{Eq. 23a}$$

$$v_c = 0.083\left(1.5 + \frac{\alpha_s d}{b_0}\right)\lambda\sqrt{f_c'} + 0.3f_{pc} + \frac{V_p}{b_0 d} \qquad \text{Eq. 23b}$$

where λ is a reduction factor for light-weight concretes, β is the aspect ratio of the column in the case of rectangular columns, f'_c is the specified compressive strength of the concrete, f_{pc} is the effective compressive stress due to prestressing, V_p is the vertical component of the effective prestress force at the critical section, b_0 is the length of the critical section, d is the effective depth, and α_s is 40 for interior columns, 30 for edge columns, 20 for corner columns.

However, it is important to note that one should be rather careful when considering these effects. The compression from prestressing in the punching shear design needs to be carefully determined. Besides the obvious factors such as creep, shrinkage, and relaxation which reduce the compression stress, one should be sure that the compression stress is acting in the vicinity of the column. The compression forces are applied at each end of the anchored prestressing tendons. If rigid structures such as shear walls are present, the compression forces may diminish within the slab and there may be no effect in the vicinity of the column. Thus, there is a risk that no beneficial effect will occur in terms of the punching strength. A certain precaution of the inclusion of this beneficial effect is thus recommended. Regarding the inclination of the tendons at the critical section, it has to be noted that the inclination of the tendons is rather difficult to check on construction sites and since they are relatively small in flat slabs, even slight changes to this angle may affect the design negatively.

Due to the limited experimental data available for prestressed slabs, ACI 318 limits the application of the prestressing effects in the punching shear design to interior slab/column connections without shear reinforcement in the slab. Consequently, the prestressing effects are not considered in the design of slabs with shear reinforcement or of slab/column connections close to slab edges. Additionally, ACI 318 sets limits on certain values:

- The minimum value for f_{pc} is 0.9 MPa. In case of smaller values, prestressing cannot be considered in the punching shear design.
- The maximum value for f_{pc} is 3.5 MPa. In case of larger values, prestressing can be considered in the punching shear design with f_{pc} = 3.5 MPa.
- The maximum value for $\sqrt{f_C'}$ is 5.8 MPa. In case of larger values, prestressing can be considered in the punching shear design with $\sqrt{f_C'}$ = 5.8 MPa.

The Shearfix software automatically respects these limitations and will provide a message if the input values are outside these boundaries.

Spacing and detailing of the shear reinforcement

The Shearfix software considers the following detailing rules:

- The distance to the first stud is predefined as 0.5d but a value between 0.35 d and 0.5 d can be chosen by the user.
- The distance between the studs within one rail is the minimum of 0.75d and 500 mm. If the factored shear stress exceeds $0.5\sqrt{f_c'}$, the maximum distance is limited to 0.5d.
- The maximum distance between adjacent rails is limited to 2d.
- A minimum of two rails are placed on each face of the column.

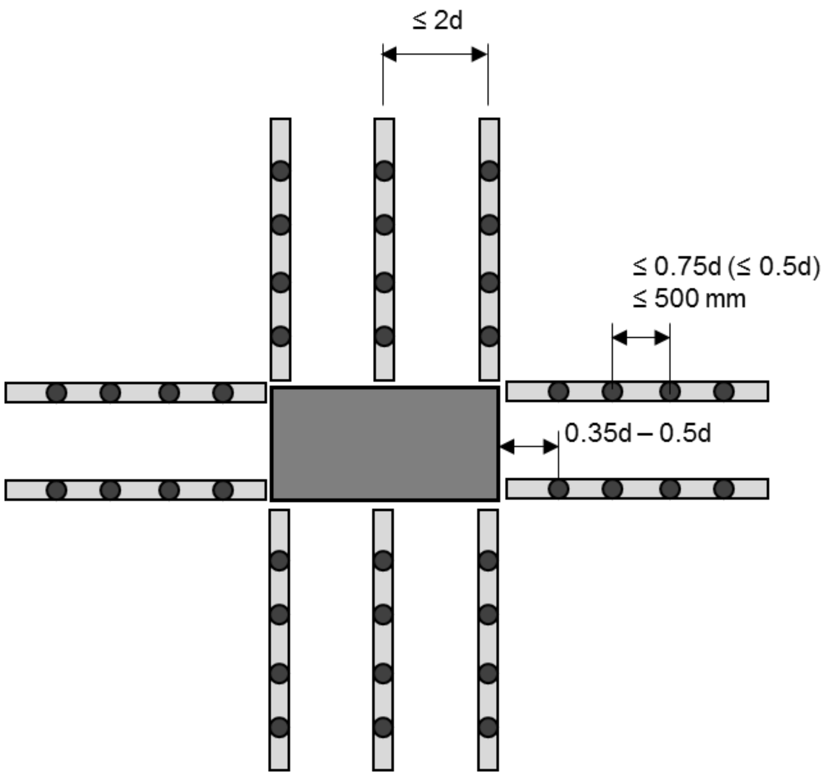

Figure 9: Detailing rules of Shearfix studs

Literature

ACI 318-19, Building Code Requirements for Structural Concrete, American Concrete Institute ACI, 628 pp. 2019

ACI 421.1R-20, Guide for Shear Reinforcement for Slabs, American Concrete Institute ACI, 33 pp., 2020